Building Structure 2
Assignment 1: Fettucini Bridge
Report

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In a group of 5, we are required to carry out precedent study of a truss bridge. Using the info obtained from the precedent study, we are required to design and construct a fettucini bridge of **600mm clear span and maximum weight of 150g**. The bridge will be tested to fail.

Other than aesthetic value, the design of the bridge must be of high efficiency, using the least material to sustain the higher load.

In this assignment, the objectives are to:

a. understand tension and compressive strength of construction materials
b. understand force distribution in a truss
c. design a perfect truss bridge which fulfils the following criterias:
i. High level of aesthetic value
ii. Minimal construction material

Any adhesive could be used for the bridge, but at the cost of weight.
Theory of truss bridge:

Truss is a formation produced by triangular components, in accordance with the truss bridge drawings, and coupled at joints known as nodes. The triangular units forming the truss are slim and straight in form. The truss bridges consist of a grouping of triangles that are manufactured from straight and steel bars, according to the truss bridge designs. The solid arms of the triangle are extended from the pier sides. The diagonal steel tubes project from the bottom and top of each pier, and assist in holding the arms in the correct position. Trusses are organized as straight elements that are connected at the ends by hinges to develop a secure arrangement.

Referenced from: David Weitzman's book "Windmills, Bridges, and Old Machines: Discovering Our Industrial Past"

On application of loads on the truss joints, forces are communicated to the truss elements. The steel truss bridge members are in compression or tension. The trusses possess a high ratio of strength to weight, and therefore are useful for being employed in truss bridges.

Trusses are also suitable for use in several other structures like roof supports and space stations. Amongst the modern bridges, truss bridges are considered to be included in the older kinds. The famous truss bridges are relatively inexpensive due to effective utilization of the bridge materials. The truss bridge designs are an important factor in architecture.
Types of truss bridges:

**Howe Truss**

The Howe truss consists of diagonal and vertical elements. The diagonal elements slope towards the bridge center in an upward direction. The vertical elements are under tension. This is a rare type of truss bridge originally patented in 1840 by William Howe. Famous examples are the Jay Bridge in New York and the Sandy Creek Covered Bridge in Missouri.

**Lenticular Truss**

This kind of bridge uses a lens-shape truss. An upper arch curves up and then down, and a lower one curves down and then up. The two arches meet at the same end points. Examples of this bridge include the Royal Albert Bridge in UK and the Smithfield Street Bridge in Pennsylvania.
Types of truss bridges:

**Bowstring Arch Truss**
This bridge was patented in 1840 by S. Whipple. The main characteristic here is that the vertical loads on the thrust arches are transmitted along the arc path. At the end of the arch, the thrust is resolved into vertical and horizontal components.

**Pegram Truss**
In this bridge, the upper chords are all of equal length and the lower chords are longer than the corresponding upper chord. Because of this difference, each panel is not square. This truss design was patented by George H. Pegram in 1885. Only ten Pegram bridges remain in the United States, and seven of them are in Idaho.

There are more than 20 designs of truss bridges catering for different types of applications and aesthetics.
The truss bridge selected for our project comes from the Warren truss. The Warren truss consists of longitudinal members joined only by angled cross-members, forming alternately inverted equilateral triangle-shaped spaces along its length, ensuring that no individual strut, beam, or tie is subject to bending or torsional straining forces, but only to tension or compression.

This configuration combines strength with economy of materials and can therefore be relatively light. The girders being of equal length, it is ideal for use in prefabricated modular bridges. It is an improvement over the Neville truss which uses a spacing configuration of isosceles triangles.

127th Street Bridge at Cook County, Illinois

This bridge is one of a number of late example metal truss bridges from the 1950s-1970s found on the Little Calumet River and Calumet Sag Channel. However, this 1968 bridge stands out for its heavy skew which gives it a very distinctive appearance not found in any of the other bridges on these waterways.
The tracing of the bridge from plans reveals that many elements of triangles are used to form the bridge. It provides many pivots where the force can be distributed.
A. Materials

The main material used is Fettuccine. It is a type of pasta popular in Roman cuisine. It is a flat thick noodle made of egg and flour (usually one egg for every 100 g of flour), wider than but similar to the tagliatelle typical of Bologna. The pasta used for this experiment is dried type, not the fresh kind.

Strength based on Brand of pasta

Different brands of pastas have different types of strength and rigidity. The following is the max loads being able to be loaded by each brand before breaking. All pastas used are dried and

a. San Remo: 110g

b. Kimball: 70g

c. Topvalu (Aeon): 80g

d. Tesco: 70g

Characteristics

Ultimate tensile strength ~2000 psi
Stiffness (Young’s modulus) E ~10,000,000 psi
B. Methodology

As pasta has a small area of contact, it is impossible to use nails or buttplates to fasten the joints together. Thus, a wide range of glues and adhesives are used in assembling the bridge.

• **Glues:**

1) **White glue**: Not good. Since it’s water based, the spaghetti is softened by the glue. Glue joints take forever to dry. Once dry, joints are not very strong.

2) **Model airplane glue**: So so. Dries relatively quickly but is slightly flexible when dry. Glue joints should be rigid.

3) **Hot-melt plastics (glue guns)**: Easiest to use, but joints far too flexible.

4) **Epoxy**: Best solution--especially the 5 minute kind. Creates rigid joints. Is messy. Requires careful mixing.

5) **Super Glue**: Best solution, Creates rigid joints. Is messy like epoxy.
Design and construction ideas:

1) Triangles are the best method of construction as there are no bending moments in triangular elements.

2) Taller is better: note loads on these two structures.

3) For economy of time, joints should be “overlaid” not “butted”. Butt joints require careful sizing. Overlaid joints do not. Excess material may be cut off after assembly.

4) Maximum area of contact should be emphasized on joints, for instance, vertical members may be turned with the flat face facing outwards the sides of the bridge to increase area and effective tensile strength.

5) Horizontal bracing are kept to a minimum length to avoid excessive weight.

6) Major members like the span and the top span are layered to keep the members stiff.
Overall in our experimentation, we tested 2 12.5 cm mockup bridges to determine the final shape of our design. After that, it is followed by a rough test of a full scale bridge at 74.2 cm of span length. Once done, we proceeded to the final model.

**Bridge tests**

Type 1 -1: 12.5 cm bowstring pulling cable bridge.

Specs:

Span: 12.5 cm

Weight withstood: 2kg

Weight of bridge estimated: 50g

Efficiency: 0.8%
Type 1 - 2: 12.5cm Warren Truss Bridge.
Specs:
Span: 12.5 cm
Weight withstood: 2kg
Weight of bridge estimated: 80g
Efficiency: 0.5%
Type 2: Warren Truss Bridge.

Side note: Even though the Bowstring was better in efficiency, it was clearly not possible to bend the pasta in lengths longer than 24cm for the top span. The construction was difficult so therefore the team chose the 2\textsuperscript{nd} most effective option which is the Warren truss bridge.

Specs:
Span: 74.2 cm
Weight withstood: 3kg
Weight of bridge estimated: 100g
Efficiency: 0.9\%
Topic 5: Testing and Verdict

Final bridge design – Perspectives and 3D rendering
Topic 5: Testing and Verdict

Final bridge design - Elevations

Top Elevation

Side Elevation

Front Elevation

Isometric
Final bridge design - Force analysis

**Legend:**
- Green – Compression \( \rightarrow \langle \rightarrow \)
- Red – Tension \( \leftrightarrow \rightarrow \)

Joint F
\[ \Sigma F_x = 0 \quad \rightarrow (1) \quad F_{fg} + F_{fq} \cos 15^\circ = 0 \]
\[ \Sigma F_y = 0 \quad \rightarrow (2) \quad F_{fq} \cos 75^\circ - 25N = 0 \]
\[ F_{fq} = 96.59 \text{ N} \]

(2) \( \rightarrow \) (1) \[ F_{fg} = -93.3 \text{ N} \]

Joint Q
\[ \Sigma F_x = 0 \quad \rightarrow (1) -96.59 \cos 75^\circ + \]
\[ F_{qr} \cos 75^\circ = 0 \]
\[ \Sigma F_y = 0 \quad \rightarrow (2) \text{ (No external force)} \]
\[ F_{qr} = 0 \]

Since angle is constant, force is same through the span.

Joint G
\[ \Sigma F_x = 0 \quad \rightarrow (1) -93.3N + F_{gh} = 0 \]
\[ \Sigma F_y = 0 \quad \rightarrow (2) \text{ (No external force)} \]
\[ F_{gq} = 0 \]
Process of constructing the final bridge

Step 1
A cad copy of the bridge is printed on to A2 paper to 1:1 scale. After that, a layer of baking paper is over the cad plan to avoid glue from sticking to the paper. This is because baking paper is layered with oil which doesn’t encourage mixture water.

Step 2
The parts of the bridge are cut into sections using the CAD as a guide. The parts are glued on one by one using super glue. If possible, the paper is separated from the glued parts before moving to the next part to ease production.
Step 3
Finally, the horizontal Bracing is produced using the 3d model drawn on sketchup as a guidance. Minimal members are placed in to save weight.
Topic 5: Testing and Verdict

**Final results**

**Spec of bridge**

Weight withstood:
5kg

Overall weight:
110g

Span:
74.2cm

Efficiency:
2.27%

Compared to the previous bridges, the final did held a better record. But it was not able to carry a lot of weight due to the low stiffness of the bridge on the base.
Overall the bridge worked the best when it has triangular elements. The truss bridge has aesthetic values of its own, being very pleasing to look at and yet still practical. Bridges like these should be maintained in the future.
Topic 7 : References

- [http://www.slideshare.net/fazirahabdulghafar/calculating-truss-forces](http://www.slideshare.net/fazirahabdulghafar/calculating-truss-forces)
Topic 8: Appendix
Case Study 1

Joint A:
\[
\sum F_x = 0 \quad -1 \quad F_{Ay} + 250 \text{ kN} + 100 \sin 53.13^\circ = 0 \\
F_{Ax} = -330 \text{ kN} \quad -2
\]
\[
\sum F_y = 0 \quad -2 \quad F_{Ay} + F_{Iy} + 150 \text{ kN} - 50 \text{ kN} \\
100 \cos 53.13^\circ - (20 \text{ kN}) = 0
\]
(As point I is a roller joint, external force of \( F_{Iz} \) is ignored.)

Moment at point A - sum all external forces
\[
\sum M_A = 0 \quad -3 \quad F_{Ay} (1.4) + F_{Ax} (0.5) + 150 \text{ kN} (0.4) - (100 \cos 53.13^\circ) \\
(3.7) E - (100 \sin 53.13^\circ) (0) - 50 (1.1) = 0
\]

1 \rightarrow 3
\[
F_{Ay} (1.4) + (-330) (0.5) + 60 - 42 - 55 = 0 \\
F_{Ay} = 144.286 \text{ kN} \quad -3
\]

3 \rightarrow 2
\[
144.286 + F_{Iy} + 150 - 50 - 100 \cos 53.13^\circ - 120 = 0 \\
F_{Iy} = 64.286 \text{ kN}
\]
Joint E (solving internal forces onwards)

\[ \sum F_x = 0 \rightarrow -120kN + F_{PE} \sin 51.34^\circ = 0 \]
\[ \sum F_y = 0 \rightarrow -250kN - F_{PE} - F_{PE} \cos 51.34^\circ = 0 \]

1. \[ F_{PE} = 153.675kN \]
2. \[ F_{FE} = 154kN \]

Joint D

\[ \sum F_x = 0 \rightarrow F_{CO} + 153.675 \sin 38.86^\circ = 0 \]
\[ \sum F_y = 0 \rightarrow 150 + F_{DF} - 153.675 \cos 38.86^\circ = 0 \]

1. \[ F_{CO} = -96.419kN \]
2. \[ F_{DF} = -30.336kN \]

Joint F

\[ \sum F_x = 0 \rightarrow -154 + F_{GF} - F_{CF} \sin 20.96^\circ = 0 \]
\[ \sum F_y = 0 \rightarrow 30.336 + F_{CF} \cos 20.96^\circ = 0 \]

1. \[ F_{CF} \cos 20.96^\circ = -30.336 \]
2. \[ F_{CF} = -35.376kN \]
3. \[ F_{GF} = 135.801kN \]
Joint C

\[ F_{HC} = 102.12 \text{ kN} \]
\[ F_{MC} = 96.419 \text{ kN} \]
\[ F_{CH} = 38.095 \text{ kN} \]
\[ F_{CG} = 60 \text{ kN} \]

\[ \sum F_{x_c} = 0 \quad \rightarrow \quad 96.419 + F_{HC} + F_{CH} \sin 38.86^{\circ} - 35.376 \cos 59.09^{\circ} = 0 \]
\[ \sum F_{y_c} = 0 \quad \rightarrow \quad -60 \text{ kN} + F_{CH} \cos 38.86^{\circ} + 35.376 \cos 30.96^{\circ} = 0 \]

\[ F_{HC} = -102.12 \text{ kN} \]

\[ \sum F_{x_c} = 0 \quad \rightarrow \quad 96.419 + F_{HC} + 38.095 \sin 38.86^{\circ} - 35.376 \cos 59.04^{\circ} = 0 \]

Joint B
\[ F_{AB} = 102.12 \text{ kN} \]
\[ F_{BC} = 102.12 \text{ kN} \]

\[ \sum F_{y_b} = 0 \quad \rightarrow \quad F_{BH} = 0 \]
\[ \sum F_{x_b} = 0 \quad \rightarrow \quad 102.12 + F_{AB} = 0 \]
\[ F_{AB} = -102.12 \text{ kN} \]
\[ \sum F_{xH} = 0 - 1 \]
\[-50 \text{ kN} - 38.095 \text{ kN} \sin 51.34^\circ + \]
\[ F_{AH} \cos 30.96^\circ = 0 \]
\[ \sum F_{yH} = 0 - 2 \]
\[ 55.801 - 38.095 \cos 51.34^\circ - F_{AH} \sin 30.96^\circ \]
\[ + F_{IH} = 0 \]

1. \[ F_{AH} = 93 \text{ kN} \]

2. \[ F_{IH} = 15.840 \text{ kN} \]

Resolving forces of joint A and I:

Joint A:
\[ \sum F_{yA} = 0 - 1 \]

144.286 + \[ F_{yA} = F_{AH} \cos 30.96^\circ = 0 \]

2. \[ \sum F_{xA} = 0 - 2 \]

\[ F_{AB} - 330 \text{ kN} - F_{AH} \sin 30.96^\circ = 0 \]

1. \[ 144.286 + F_{yA} - (93) \cos 30.96^\circ = 0 \]
\[ F_{yA} = -64.536 \text{ kN} \]
Overall conclusion:

\[
P_{AY} = 164.286 \text{kN}
\]

\[
P_{AB} = 33.04 \text{kN}
\]

\[
P_{AH} = 9.97 \text{kN}
\]

\[
P_{BH} = 0
\]

\[
P_{AE} = 64.536 \text{kN}
\]

\[
P_{AE} = 15.940 \text{kN}
\]

\[
P_{AE} = 55.296 \text{kN}
\]

\[
P_{AE} = 133.801 \text{kN}
\]

\[
P_{AE} = 159.4 \text{kN}
\]

\[
P_{CD} = 96.417 \text{kN}
\]

\[
P_{EF} = 153.675 \text{kN}
\]
CASE STUDY 2 (CHUAH CHU KING 0303769)

\[ \text{Determine Perfect Truss:} \]

\[ 2J = m + 3 \], where \( J \) = no of joints
\( m \) = no of members.

\[ 2(9) = (15) + 3 \]
\[ 18 = 18 \]

This is a perfect truss.

\[ \text{Determine the reaction forces:} \]

\[ \text{\( F_{x}=0 \)} \]

\[ F_{x1} + F_{x2} + 250 + (100 \times \frac{8}{10}) = 0 \]
\[ (F_{x1} + F_{x2}) = -330 \text{ KN} \] ... (i)

\[ \text{\( F_{y}=0 \)} \]

\[ F_{y1} + 150 + (50 + 100 \times \frac{5}{10} + 120) = 0 \]
\[ F_{y1} = 80 \text{ KN} \]

Therefore, vertical force at point \( F_{y1} \), is 80 KN towards.

\[ \text{\( E_M=0 \), where \( E \) = moment} \]

Taking moment of point (1)
\[ (F_{x1} \times 0.5) + (50 \times 0.3) + (100 \times 0.5 \times 0.7) - (150 \times 1) + (120 \times 1.4) = 0 \]
\[ 0.5 F_{x1} + 15 + 42 - 150 + 168 = 0 \]
\[ 0.5 F_{x1} = 75 \]
\[ F_{x1} = 150 \text{ KN} \] ... (ii)

Horizontal resultant force at point 1 \& 2 is 180 KN \& 150 KN respectively towards left.
Determine forces in each member.

\[ F_0 = 100 \text{ kN} \]

\[ 0.5 = \frac{F_0}{100} \]

\[ m = 5 \]

\[ F_0 = 50 \text{ kN} \]

\[ F_0 \cos 51.3 = 154 \text{ kN} \]

\[ F_0 \sin 51.3 = 120 \text{ kN} \]

\[ F_0 = 153.8 \text{ kN} \]

\[ F_0 = 154 \text{ kN} \]

\[ F \cos 51.3 = 96.3 \text{ kN} \]

\[ F \sin 51.3 = 38.4 \text{ kN} \]

\[ F = 137.8 \text{ kN} \]

\[ F = 172 + (100 \times \frac{9}{10}) \]

\[ F = 60 \text{ kN} \]

\[ F_0 = 60 \text{ kN} \]
\( \sum F = 0 \)

\[ 140 - F_\theta \theta = 0 \]

\[ F_\theta \theta = 140 \text{ KN} \]

\( \sum M = 0 \)

\[ 198 \times 2 + 67.2 \cos 51.3 - 114 \times 1 \cos 59.0 \cos 59.0 \cos 59.0 - 96.2 \]

\[ F_\theta \theta = 186.8 \text{ KN} \]

\( \sum y = 0 \)

\[ F_\theta \theta + 67.2 \sin 51.3 + 186.8 \sin 59.0 - 162.6 \text{ KN} \]
Case Study 3

Determine Perfect Truss

\[ 2J = M + 3, \text{ where } J = \text{ no of joints} \]
\[ M = \text{ no of members} \]

\[ 2(9) = (15) + 3 \]
\[ 18 = 18 \]

\[ \therefore \text{ This is the perfect truss.} \]

Determine The Reaction Forces / Contact Force

\[ \Sigma F_x = 0 \]
\[ F_{HX} + F_{IX} + 250 + 80 = 0 \]
\[ F_{HX} + F_{IX} + 330 = 0 \]
\[ F_{HX} + F_{IX} = -330 \quad (1) \]

\[ \Sigma M = 0, \text{ where } r = \text{ moment} \]
\[ (FH \times 6.5) + (50 \times 0.8) + (-150 \times 1.0) + (60 \times 0.7) + (120 \times 1.4) = 0 \]
\[ 0.5F_{HX} + 15 + (-150) + 42 + 168 = 0 \]
\[ 0.5F_{HX} + 75 = 0 \]
\[ 0.5F_{HX} = -75 \]
\[ F_{HX} = -150 \text{ kN} \quad \text{put into (1)} \]

\[ \therefore \text{ Horizontal reaction force at point H is 150 kN to the left} \]
\[ F_{HX} + F_{IX} = -330 \]
\[ -150 + F_{IX} = -330 \]
\[ F_{IX} = -180 \text{ kN} \]
Determine the forces in each member.

\[ EF_x = 0 \]
\[ 250 + (-FAc) + (-FAB \sin 38.7^\circ) = 0 \]
\[ 250 - FAC - 153.8 \sin 38.7^\circ = 0 \]
\[ FAC = 153.8 \text{ kN} \uparrow \quad \text{(Tension)} \]

\[ EF_y = 0 \]
\[ FAB \cos 38.7^\circ + (-120) = 0 \]
\[ FAB \cos 38.7^\circ - 120 = 0 \]
\[ FAB \cos 38.7^\circ = 120 \]
\[ FAB = 153.8 \text{ kN} \uparrow \quad \text{(Tension)} \]

\[ FB_D = 150 \text{ kN} \]
\[ 153.8 \cos (51.3^\circ) = 96.2 \text{ kN} \]

\[ EF_x = 0 \]
\[ 96.2 + (-FBD) = 0 \]
\[ FBD = 96.2 \text{ kN} \uparrow \quad \text{(Tension)} \]

\[ 153.8 \sin (51.3^\circ) = 120 \text{ kN} \]

\[ EF_y = 0 \]
\[ 150 + (-FBC) + (-120) = 0 \]
\[ 150 - FBC - 120 = 0 \]
\[ 150 - 120 = FBC \]
\[ FBC = 30 \text{ kN} \uparrow \quad \text{(Tension)} \]

\[ FBC \] \[ \theta = 59^\circ \]
\[ \tan \theta = 0.5 \]
\[ 0.5 \]
\[ 0.4 \]
\[ \theta = 59^\circ \]

\[ EF_y = 0 \]
\[ FBC - (35) \sin 59^\circ = 0 \]
\[ FBC = 30 \text{ kN} \uparrow \quad \text{(Tension)} \]

\[ EF_x = 0 \]
\[ -FEC + (35) \cos 59^\circ + FCA = 0 \]
\[ -172 + 18 + FCA = 0 \]
\[ FCA = 154 \text{ kN} \uparrow \quad \text{(Tension)} \]
\( \Sigma F_y = 0 \)
30 + \( F_{DC} \sin 59^\circ = 0 \)
\( F_{DC} = -35 \text{ kN} \) (compression)

\( \Sigma F_x = 0 \)
- \( F_{FD} + F_{DB} + (-35) \cos 59^\circ = 0 \)
- 78 + \( F_{DB} - 18 = 0 \)
\( F_{DB} = 96 \text{ kN} \) (Tension)

\( \Sigma F_y = 0 \)
\( F_{DE} + (38.4) \sin 31.3^\circ - 50 = 0 \)
\( F_{DE} = 30 \text{ kN} \) (Tension)

\( \Sigma F_x = 0 \)
- \( F_{GE} + F_{EC} + 80 - (38.4) \cos 51.3^\circ = 0 \)
- 228 + \( F_{EC} + 80 - 24 = 0 \)
\( F_{EC} = 172 \text{ kN} \) (Tension)

\( \Sigma F_y = 0 \)
- \( F_{FE} \sin 51.3^\circ + F_{FG} = 0 \)
- \( F_{FE} \sin 51.3^\circ + 30 = 0 \)
\( F_{FE} = 38.4 \text{ kN} \) (Tension)

\( \Sigma F_x = 0 \)
- \( F_{HF} + F_{FD} + (38.4) \cos 51.3^\circ = 0 \)
\( F_{HF} = 38.4 \text{ kN} \) (Tension)

\( \Sigma F_X = 0 \)
- \( F_{HF} + F_{FO} + (38.4) \cos 51.3^\circ = 0 \)
- 102 + \( F_{FO} = -24 \)
\( F_{FO} = 78 \text{ kN} \) (Tension)
\[ \Sigma F_y = 0 \]

\[ F_{HG} \sin 59^\circ - 50 + F_{FG} = 0 \]

\[ F_{FG} = -30 \text{ kN} \# (\text{compression}) \]

\[ \Sigma F_x = 0 \]

\[ -180 - 93.3 \cos 59^\circ + F_{GE} = 0 \]

\[ -180 - 48 + F_{GE} = 0 \]

\[ F_{GE} = 228 \text{ kN} \# (\text{Tension}) \]

\[ \Sigma F_y = 0 \]

\[ 80 - F_{HG} \sin 59^\circ = 0 \]

\[ F_{HG} = 93.3 \text{ kN} \# (\text{Tension}) \]

\[ \Sigma F_x = 0 \]

\[ -150 + F_{HG} \cos 59^\circ + F_{HG} = 0 \]

\[ F_{HG} = 102 \text{ kN} \# (\text{Tension}) \]

\[ \Sigma F_y = 0, \ F_H = 0 \]

\[ \Sigma F_x = 0, \ F_H - 180 = 0 \]

\[ F_H = 180 \text{ kN} \# (\text{Tension}) \]
Determine Perfect Truss

1. \( 2J = mt + 3 \)
2. \( 2(9) = (15)/8 \)
3. \( 18 = 18 \)

\( \therefore \) The structure is a perfect truss.

Determine the Reaction Forces/Contact Forces

1. \( \sum F_x = 0 \)
   \[ F_{Hx} + F_{Ix} + 80 + 250 = 0 \]
   \[ F_{Hx} + F_{Ix} + 330 = 0 \]
   \[ F_{Hx} + F_{Ix} = -330 \quad \text{(1)} \]

2. \( \sum F_y = 0 \)
   \[ F_{Hy} + 150 + (-120) + (-100x\frac{1}{10}) = 0 \]
   \[ F_{Hy} - 80 = 0 \]
   \[ F_{Hy} = 80 \text{ kN} \]

\( \therefore \) Vertical reaction force at point I is 80 kN upwards.

3. \( \sum M = 0, \) moment
   \[ (F_{Hx} \times 0.5) + (50 \times 0.3) + (-150 \times 1.0) + (60 \times 0.7) + (120 \times 1.4) = 0 \]
   \[ 0.5F_{Hx} - 15 - 150 + 42 + 168 = 0 \]
   \[ 0.5F_{Hx} + 75 = 0 \]
   \[ 0.5F_{Hx} = -75 \]
   \[ F_{Hx} = -150 \text{ kN} \quad \text{(2)} \]

\( \therefore \) Horizontal reaction force at point I is -150 kN towards left.

Substitute eq. (2) into (1):

\[ F_{Hx} + F_{Ix} = -330 \]
\[ -150 + F_{Ix} = -330 \]
\[ F_{Ix} = -180 \text{ kN} \]
Determine internal forces in each member

Joint A:

\[ \tan \theta = \frac{0.4}{0.5} \]
\[ \theta = 38.7^\circ \]
\[ \Sigma x = 0 \]
\[ 250 + (-F_{AC}) + (-F_{AB} \sin 38.7^\circ) = 0 \]
\[ 250 - F_{AC} - 153.8 \sin 38.7^\circ = 0 \]
\[ F_{AC} = 153.8 \text{ kN} \]
(tension)

\[ \Sigma y = 0 \]
\[ F_{AB} \cos 38.7^\circ + (-120) = 0 \]
\[ F_{AB} \cos 38.7^\circ = 120 \]
\[ F_{AB} = 153.8 \text{ kN} \]
(tension)

Joint B:

\[ \tan \theta = \frac{0.5}{0.4} \]
\[ \theta = 51.3^\circ \]
\[ \Sigma x = 0 \]
\[ (-F_{BD}) + (F_{BA} \cos 51.3^\circ) = 0 \]
\[ -F_{BD} + 96.2 = 0 \]
\[ F_{BD} = 96.2 \text{ kN} \]
(tension)

\[ \Sigma y = 0 \]
\[ 150 + (-F_{BC}) + (-F_{BA} \sin 51.3^\circ) = 0 \]
\[ 150 - F_{BC} - 153.8 \sin 51.3^\circ = 0 \]
\[ 150 - F_{BC} - 120 = 0 \]
\[ F_{BC} = 30 \text{ kN} \]
(tension)

Joint C:

\[ \tan \theta = \frac{0.5}{0.3} \]
\[ = 59^\circ \]
\[ \Sigma y = 0 \]
\[ F_{BC} + F_{CD} \sin 59^\circ = 0 \]
\[ 30 + F_{CD} \sin 59^\circ = 0 \]
\[ F_{CD} \sin 59^\circ = -30 \]
\[ F_{CD} = -35 \text{ kN} \]
(compression)

Joint D:

\[ \tan \theta = \frac{0.5}{0.4} \]
\[ \theta = 51.3^\circ \]
\[ \Sigma x = 0 \]
\[ F_{CD} \cos 59^\circ + (-F_{DE}) + (-F_{DF}) = 0 \]
\[ -35 \cos 59^\circ - (-35 \cos 59^\circ) + 96.2 - F_{DF} = 0 \]
\[ -18 + 24 + 96.2 = F_{DF} \]
\[ F_{DF} = 102.2 \text{ kN} \]
(tension)

\[ \Sigma y = 0 \]
\[ F_{CD} \sin 59^\circ + F_{DE} - F_{DG} \sin 51.3^\circ = 0 \]
\[ 35 \sin 59^\circ - 60 - F_{DG} \sin 51.3^\circ = 0 \]
\[ 30 - 60 - F_{DG} \sin 51.3^\circ = 0 \]
\[ F_{DG} \sin 51.3^\circ = -30 \]
\[ F_{DG} = -38.4 \text{ kN} \]
(compression)
Joint E:

\[ \Sigma x = 0 \]
- \( F_{EG} + F_{CE} + (100 \times \frac{8}{10}) = 0 \)
- \( F_{EG} + 171.8 + 80 = 0 \)
\( F_{EG} = 251.8 \text{ kN (tension)} \)

\[ \Sigma y = 0 \]
- \( F_{DE} + (-60) = 0 \)
\( F_{DE} = 60 \text{ kN (tension)} \)

Joint F:

\[ \tan \theta = \frac{0.5}{0.3} \]
\[ \theta = 59.6^\circ \]
\[ \Sigma x = 0 \]
- \( F_{FH} + F_{DF} + (F_{FI} \cos 59) = 0 \)
- \( F_{FH} + 102.2 - (43.3 \cos 59) = 0 \)
- \( F_{FH} + 102.2 + 48 = 0 \)
\( F_{FH} = 150.2 \text{ kN (tension)} \)

\[ \Sigma y = 0 \]
- \( F_{EG} + (F_{FI} \sin 59) = 0 \)
- \( -80 = F_{FI} \sin 59 \)
\( F_{FI} \sin 59 = 80 \)
\( F_{FI} = 93.3 \text{ kN (compression)} \)

Joint G:

\[ \tan \theta = \frac{0.5}{0.4} \]
\[ \theta = 51.3^\circ \]
\[ \Sigma x = 0 \]
\( F_{EG} + (-F_{GI}) + (-F_{GI} \cos 51.3) = 0 \)
\( 251.8 - F_{GI} - (38.4 \cos 51.3) = 0 \)
\( 251.8 - F_{GI} - 24 = 0 \)
\( F_{GI} = 227.8 \text{ kN (tension)} \)

Joint H:

\[ \Sigma x = 0 \]
- \( -150 + F_{FH} = 0 \)
\( F_{FH} = 150 \text{ kN (tension)} \)

\[ \Sigma y = 0 \]
- \( F_{HI} = 0 \)
\( F_{HI} = 0 \)
Conclusion:

\[\begin{align*}
150kN & \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow 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CASE STUDY 5

Determine Perfect Truss:

2J = m + 3, where J = no. of joints
m = no. of members.

2(9) = (15) + 3
18 = 18 (balanced)

.: This structure is a perfect truss.

Determine the Reaction Forces:

\[ \sum F_x = 0 \]

\[ F_{ax} + F_{bx} + 250 + (100 \times \frac{8}{10}) = 0 \]

\[ F_{ax} + F_{bx} + 330 = 0 \]

\[ F_{ax} + F_{bx} = -330 \text{ KN} \quad (\text{1}) \]

\[ \sum F_y = 0 \]

\[ F_{by} + 150 + (-50) + (-100 \times \frac{6}{10}) + (120 \times 1.4) = 0 \]

\[ F_{by} - 80 = 0 \]

\[ F_{by} = 80 \text{ KN} \]

.: Vertical resultant force at point B is 80KN upwards.

\[ \sum M = 0 \] moment
Taking moment at point B,

\[ (F_{ax} \times 0.5) + (-150 \times 1.0) + (50 \times 0.3) + (100 \times \frac{6}{10} \times 0.7) + (120 \times 1.4) = 0 \]

\[ 0.5 F_{ax} = 150 + 15 + 42 + 168 = 0 \]

\[ 0.5 F_{ax} + 75 = 0 \]

Horizontal \( F_{ax} = -150 \text{ KN} \) put into (1)

.: Resultant force at point A is 150KN towards left.

\[ F_{ax} + F_{bx} = -330 \text{ KN} \]

\[ -150 + F_{bx} = -330 \]

\[ F_{bx} = -180 \text{ KN} \]

.: Horizontal resultant force at point B is 180KN towards left.
**Determine Forces in Each Members**

Taking upwards and right = +ve
downwards and left = -ve

### A

- **FAB**
  - \( \tan \theta_1 = \frac{0.5}{0.4} \)
  - \( \theta_1 = 51.3^\circ \)

\[ \sum y = 0 \]
\[ +20 - FAB \sin 51.3^\circ - 120 = 0 \]
\[ FAB = 154 \text{ KN} \]

\[ \sum x = 0 \]
\[ 250 - FAC - 154 \cos 51.3^\circ = 0 \]
\[ FAC = 153.7 \text{ KN} > 154 \text{ KN} \]

### B

- **FBC**
  - \( \sum x = 0 \)
  - \( 154 \cos 51.3^\circ - FBD = 0 \)
  - \( FBD = 96.3 \text{ KN} > 96 \text{ KN} \)

\[ \sum y = 0 \]
\[ 150 - FBC - 154 \sin 51.3^\circ = 0 \]
\[ FBC = 30 \text{ KN} \]

### C

- **Fco**
  - \( \tan \theta_2 = \frac{0.5}{0.3} \)
  - \( \theta_2 = 59^\circ \)

\[ \sum x = 0 \]
\[ 154 - 35 \cos 59^\circ - Fce = 0 \]
\[ Fce = 106 \text{ KN} \]

\[ \sum y = 0 \]
\[ 30 + Fco \sin 59^\circ = 0 \]
\[ Fco = -35 \text{ KN} \]
\[ \begin{align*}
\Sigma x &= 0 \\
96 \cos 59 + 35 \cos 59 - F_{DE} &= 0 \\
\Sigma y &= 0 \\
35 \sin 59 - 60 - F_{DG} \sin 51.3 &= 0 \\
F_{DG} &= -38.4 \text{ kN} \\
F_{DG} &= -38 \text{ kN} \\
\end{align*} \]

\[ \begin{align*}
\Sigma x &= 0 \\
136 - F_{EG} &= 0 \\
F_{EG} &= 136 \text{ kN} \\
\Sigma y &= 0 \\
F_{ED} = 100 (\frac{6}{10}) &= 0 \\
F_{ED} &= 60 \text{ kN} \\
\end{align*} \]

\[ \begin{align*}
\Sigma x &= 0 \\
137.8 - F_{FH} - 35 \cos 59 &= 0 \\
F_{FH} &= 126 \text{ kN} \\
\Sigma y &= 0 \\
-20 - F_{FI} \sin 59 &= 0 \\
F_{FI} &= -23.3 \text{ kN} \\
\end{align*} \]

\[ \begin{align*}
\Sigma x &= 0 \\
316 - 38.4 \cos 51.3 + F_{FI} &= 0 \\
F_{FI} &= -192 \text{ kN} \\
\Sigma y &= 0 \\
F_{GF} - 50 - 38.4 \sin 51.3 &= 0 \\
F_{GF} &= 20 \text{ kN} \\
\end{align*} \]
\[ Ex = 0 \]
\[ -(180 + 23 \cos 59) + 192 = 0 \]
\[ -192 + 192 = 0 \]
\[ \therefore \text{Solved horizontal forces for both internal and external.} \]

\[ Ey = 0 \]
\[ 80 + F_{IH} - 23 \sin 59 = 0 \]
\[ F_{IH} = -60.3 \, \text{KN} \]
Conclusion Case Study

Overall it seems case 1 and 2 tend to have reduced loads by the hinges and also have one non-load bearing member. It is then suggested that case 1/2 are the strongest truss due to the lower forces being spread through the trusses. A few members can be used for this (these case) to save weight member BH (refer case 1) don’t bear any weight.